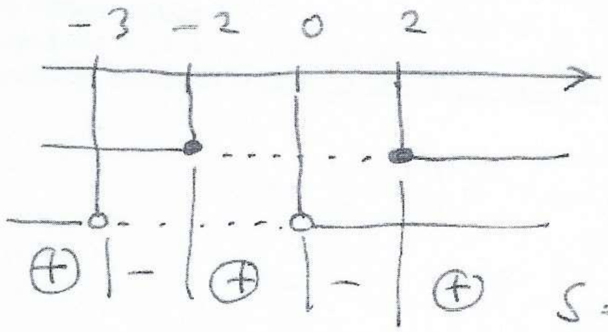
1a) a) $\dfrac{x^2-4}{x^2+3x} \geq 0$

$x^2-4 \geq 0 \Rightarrow x \leq -2 \vee x \geq 2$

$x^2+3x > 0 \Rightarrow x < -3 \vee x > 0$



$S = (-\infty, -3) \cup [-2, 0) \cup [2, +\infty)$

ok

b) $\dfrac{(1-x^2)(5x+2)}{(6-x)^2(x^2-7x+10)} \leq 0$

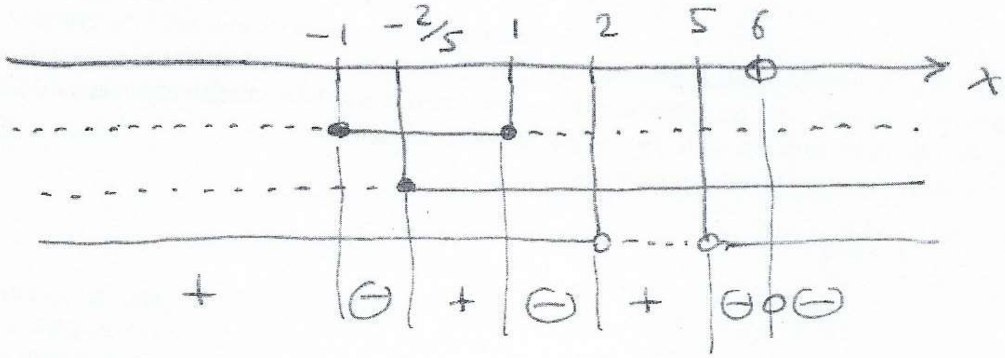
$1-x^2 \geq 0 \Rightarrow -1 \leq x \leq 1$

$5x+2 \geq 0 \Rightarrow x \geq -\frac{2}{5}$

$(6-x)^2 > 0 \Rightarrow x \neq 6$

$x^2-7x+10 > 0$ $\quad x_{1,2} = \dfrac{7 \pm \sqrt{49-40}}{2} \begin{cases} 5 \\ 2 \end{cases}$

$\Rightarrow x < 2 \vee x > 5$



$S = [-1, -\frac{2}{5}] \cup [1, 2) \cup (5, 6) \cup (6, +\infty)$

ok 1

1/1

c) $\sqrt[3]{x^3-4} = x-1$

$\cancel{x^3}-4 = \cancel{x^3}-3x^2+3x-1$

$\cancel{3}x^2 - \cancel{3}x - \cancel{3} = 0 \Rightarrow x^2 - x - 1 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$ ok

d) $\sqrt{2x+1} = 3x-2$

$\left\{\begin{array}{l} 2x+1 \geqslant 0 \Rightarrow x \geqslant -\frac{1}{2} \\ 3x-2 \geqslant 0 \Rightarrow x \geqslant \frac{2}{3} \end{array}\right| \Rightarrow x \geqslant \frac{2}{3}$

$2x+1 = (3x-2)^2$

$2x+1 = 9x^2-12x+4$

$9x^2-14x+3=0$

$x_{1,2} = \frac{7 \pm \sqrt{49-27}}{9} = \begin{cases} \frac{7+\sqrt{22}}{9} \approx 1{,}3 \\ \frac{7-\sqrt{22}}{9} \approx 0{,}3 \end{cases}$

$\Rightarrow S = \left\{\frac{7+\sqrt{22}}{9}\right\}$ ok

1/ e) $\sqrt{3x+1} < x+7$

$$\begin{cases} 3x+1 \geq 0 & \Rightarrow & x \geq -\frac{1}{3} \\ x+7 > 0 & \Rightarrow & x > -7 \\ 3x+1 < (x+7)^2 \end{cases} \Bigg| \Rightarrow x \geq -\frac{1}{3}$$

$3x+1 < x^2+14x+49$

$x^2+11x+48 > 0$

$\Delta = 121 - 192 < 0 \Rightarrow \forall x \in \mathbb{R}$

Pertanto $S = [-\frac{1}{3}, +\infty)$ ok

f) $\sqrt{4-x} > 3x-2$

$$\begin{cases} 4-x \geq 0 \\ 3x-2 < 0 \end{cases} \vee \begin{cases} 4-x > (3x-2)^2 \\ 3x-2 \geq 0 \end{cases}$$

$$\begin{cases} x \leq 4 \\ x < \frac{2}{3} \end{cases}$$

$x < \frac{2}{3}$

$x \geq \frac{2}{3}$

$\cancel{4}-x > 9x^2-12x+\cancel{4}$

$9x^2-11x < 0$

$$\begin{cases} 0 < x < \frac{11}{9} \\ x \geq \frac{2}{3} \end{cases}$$

$\Rightarrow S = (-\infty, \frac{11}{9})$ ok

3

1/ g) $|x^2-16|=12$

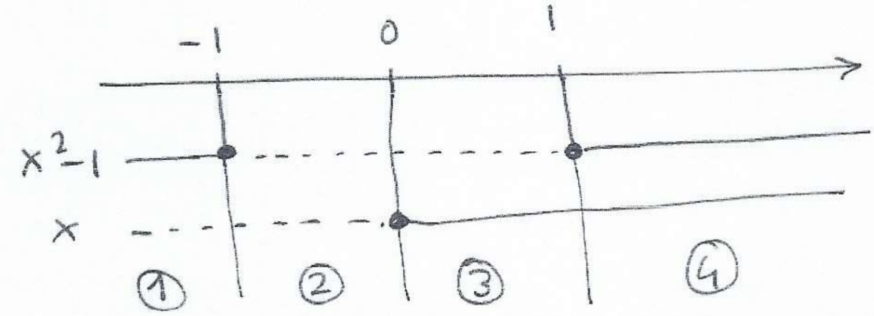
$$x^2-16=12 \quad \vee \quad x^2-16=-12$$

$$x^2=28 \qquad\qquad x^2=4$$

$$x=\pm 2\sqrt{7} \quad \vee \quad x=\pm 2$$

$$S=\{-2\sqrt{7};\ -2;\ 2;\ 2\sqrt{7}\}$$ ok

h) $|x^2-1| = -|x|+x+2$



$-1<x<0 \qquad -x^2+1 = x+x+2 \Rightarrow x^2+2x+1=0$

$\Rightarrow \cancel{x=-1}$ N.A.

$x \leq -1 \qquad x^2-1 = x+x+2$

$$x^2-2x-3=0$$

$$x_{1,2} = 1\pm\sqrt{4} = \begin{cases} \cancel{3} & \text{N.A.} \\ -1 & \text{ok} \end{cases}$$

$0 \leq x < 1 \qquad -x^2+1 = -\cancel{x}+\cancel{x}+2$

$x^2=-1$ imposs.

./.

%

$x \geq 1 \qquad x^2 - 1 = -\cancel{x} + \cancel{x} + 2$

$x^2 = 3 \qquad x = \begin{cases} \cancel{-\sqrt{3}} & \text{N.A.} \\ \sqrt{3} \end{cases}$

$\Rightarrow \; S = \{-1 ; \sqrt{3}\}$ ok

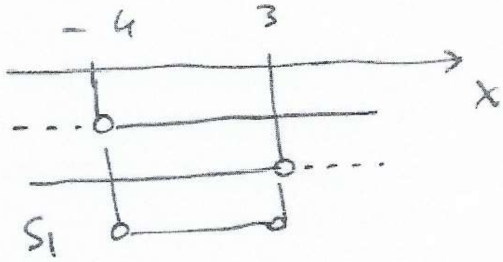
1
//

i) $\left| \dfrac{2x+1}{3-x} \right| < 1$

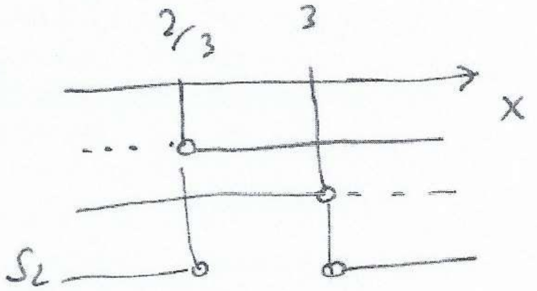
$$\begin{cases} \dfrac{2x+1}{3-x} > -1 \\[2ex] \dfrac{2x+1}{3-x} < 1 \end{cases} \qquad \begin{cases} \dfrac{2x+1+3-x}{3-x} > 0 \\[2ex] \dfrac{2x+1-3+x}{3-x} < 0 \end{cases}$$

Quindi

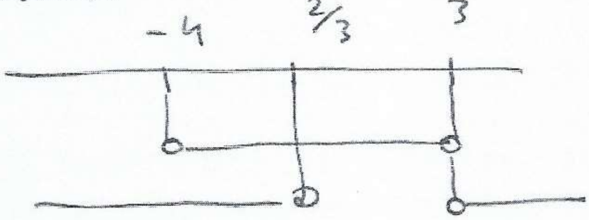
$\dfrac{x+4}{3-x} > 0 \Rightarrow$



$\dfrac{3x-2}{3-x} < 0 \Rightarrow$



Pertanto



$\Rightarrow \; S = \left(-4 ; \tfrac{2}{3}\right)$ ok

5

1/1

m) $|x+1| < 3x-4$

$x < -1$ $\quad -x-1 < 3x-4$

$-4x+3<0$

$4x-3>0 \Rightarrow x > \frac{3}{4}$ $\emptyset$

$x \geq -1$ $\quad x+1 < 3x-4$

$-2x+5<0$

$2x-5>0 \Rightarrow x > \frac{5}{2}$

$\Rightarrow S = \left(\frac{5}{2}, +\infty\right)$ ok

n) $\sqrt{2x+4} \geq |1-x|$

$2x+4 \geq (1-x)^2$

$2x+4 \geq x^2-2x+1$

$x^2-4x-3 \leq 0$

$x^2 = 2 \pm \sqrt{4+3} = \begin{cases} 2+\sqrt{7} \approx 4,\dots \\ 2-\sqrt{7} \approx -0,\dots \end{cases}$

$\Rightarrow S = \left[2-\sqrt{7}, 2+\sqrt{7}\right]$ ok

14 p) $\sqrt{2-|1+x|} \le |x+3|$

$2-|1+x| \ge 0 \Rightarrow |1+x| \le 2$

$-2 \le 1+x \le 2$

$-3 \le x \le 1$ CE

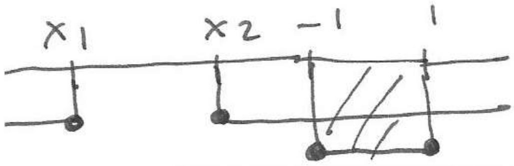
$2-|1+x| \le x^2+6x+9$

se $x \ge -1 \iff -1 \le x \le 1$

$x^2+7x+8 \ge 0$

$\Delta = 49-32 = 17$

$\Rightarrow x_{1,2} = \frac{-7 \pm \sqrt{17}}{2}$ $\begin{cases} x_1 \cong -5,6 < -1 \\ x_2 \cong -1,4 < -1 \end{cases}$



$\Rightarrow S_1 = [-1, 1]$

se $x < -1 \iff -3 \le x < -1$

$2+1+x \le x^2+6x+9$

$x^2+5x+6 \ge 0$

$x_{1,2} = \frac{-5 \pm \sqrt{25-24}}{2} = \begin{cases} -3 \\ -2 \end{cases}$

$\Rightarrow x \le -3 \vee x \ge -2$

$S_2 = [-2, -1) \cup \{-3\}$

$\Rightarrow S = S_1 \cup S_2 = [-2, 1] \cup \{-3\}$

2//

a) $\begin{cases} |2+3x| < 1 \\ 4 > \sqrt{2-x} \end{cases}$

I) $|2+3x| < 1 \Rightarrow \quad -1 < 2+3x < 1$

$-3 < 3x < -1$

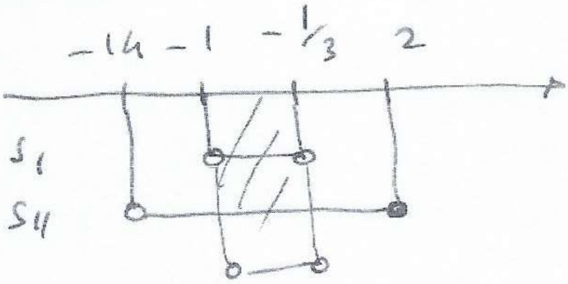
$S_I: \quad -1 < x < -\frac{1}{3}$

II) $4 > \sqrt{2-x} \qquad x \leq 2 \quad CE$

$16 > 2-x \Rightarrow x > -14$

$\Rightarrow \quad S_{II}: \ -14 < x \leq 2$

Pertanto



$\Rightarrow S = \left(-1, -\frac{1}{3}\right)$ ok

b) I) $\dfrac{5-2x-\sqrt{4-x}}{x^2+x+9} \geq 0$

$>0 \ \forall x$

$\Rightarrow \quad 5-2x-\sqrt{4-x} \geq 0$

$\sqrt{4-x} \leq 5-2x$

./.

./.

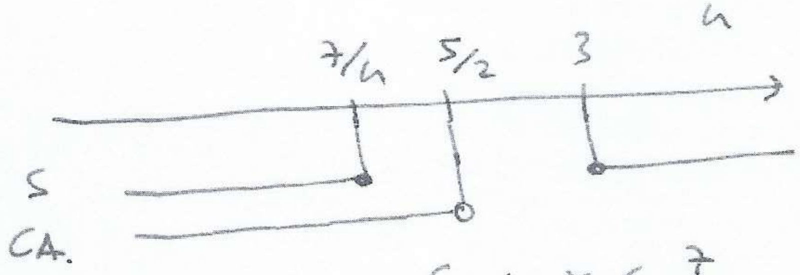
$$\begin{cases} 4-x \geq 0 \Rightarrow x \leq 4 \\ 5-2x > 0 \Rightarrow x < \frac{5}{2} \\ 4-x \leq (5-2x)^2 \end{cases} \quad \Rightarrow x < \frac{5}{2} \quad \text{C.A.}$$

$$4-x \leq 4x^2-20x+25$$

$$4x^2-19x+21 \geq 0$$

$$\Delta = 19^2-16\cdot 21 = 25$$

$$x_{1,2} = \frac{19 \pm \sqrt{25}}{8} = \begin{cases} 3 \\ \frac{7}{4} \end{cases}$$



$$S_1: x \leq \frac{7}{4}$$

II)

$$\frac{3-|4x-5|}{5+x+\sqrt{x}} < 0 \qquad \text{CE}: \ x \geq 0$$

$$\Downarrow$$

denom ≥ 5 !

Pertanto

il segno dipende soltanto dal numeratore

$$3-|4x-5| < 0 \Rightarrow |4x-5| > 3$$

%.

$$4x-5<-3 \quad \vee \quad 4x-5>3$$

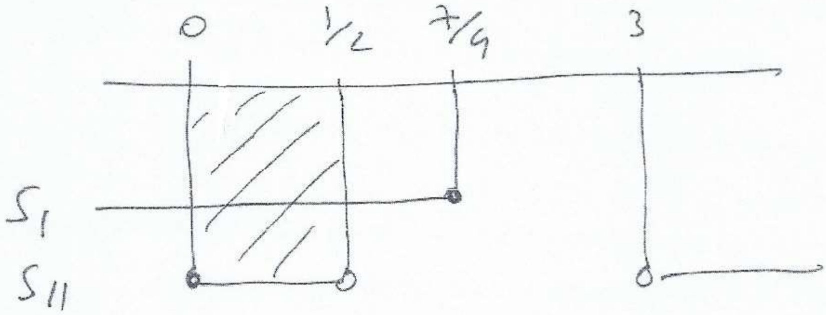
$$4x<2 \qquad\qquad 4x>8$$

$$\Rightarrow x<\frac{1}{2} \quad \vee \quad x>2$$

con $x \geq 0$ (CE)

$$\Rightarrow \quad S_{II} = \left[0, \tfrac{1}{2}\right) \cup (2, +\infty)$$

Pertanto



$$\Rightarrow \quad S = \left[0, \tfrac{1}{2}\right)$$

ok

3/ a) $\sqrt{x^2-2k} = 4$

se $k \leq 0 \Rightarrow x^2 - 2k \geq 0 \Rightarrow$ CE: $\mathbb{R}$

risolvo

$$x^2 - 2k = 16$$
$$x^2 = 16 + 2k$$
$$x_{1,2} = \pm\sqrt{16+2k}$$

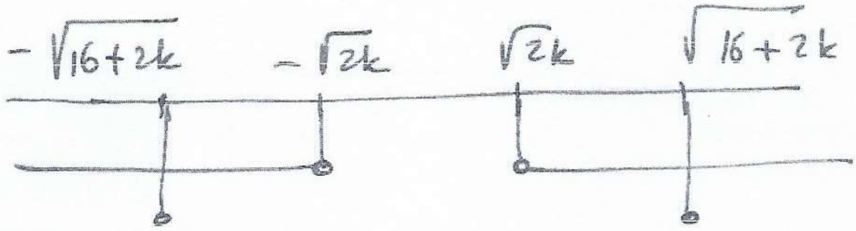
con $k \geq -8$

se $k > 0 \Rightarrow x^2 - 2k \geq 0 \Rightarrow x^2 \geq 2k$

cioè CE: $x \leq -\sqrt{2k} \vee x \geq \sqrt{2k}$

risolvo

$$x_{1,2} = \pm\sqrt{16+2k}$$



$$\Rightarrow x_{1,2} = \pm\sqrt{16+2k}$$

Pertanto

se $k < -8$ $\nexists$ sol

se $k \geq -8$ 2 soluzioni: $x_{1,2} = \pm\sqrt{16+2k}$

ok (11)

3//

b) $\sqrt{x+2} < 3k-1$

se $3k-1>0 \Rightarrow k > \frac{1}{3}$

$$\begin{cases} x+2 \geq 0 \\ x+2 < 9k^2-6k+1 \end{cases}$$

$$\begin{cases} x < 9k^2-6k-1 \\ x \geq -2 \end{cases}$$

Pertanto

se $k > \frac{1}{3} \Rightarrow -2 \leq x < 9k^2-6k-1$

cioè $S = [-2, 9k^2-6k-1)$

se $k \leq \frac{1}{3}$ $\nexists$ sol

ok

3/II c) $|2x^2+2k| = 3k$

se $k<0$ $\nexists$ sol

se $k \geq 0$

$2x^2+2k = 3k \Rightarrow 2x^2 = k$

$x = \pm\sqrt{\frac{k}{2}}$

d) $1 > k - |x|$

$|x| > k-1$

se $k-1 \leq 0$ $\forall x \in \mathbb{R}$

se $k-1 > 0$

$x < 1-k \vee x > k-1$

4/1

$$y = f(x) = \frac{\sqrt{x} - \sqrt{3-x}}{x-1}$$

$$\begin{cases} x \geq 0 \\ 3 - x \geq 0 \Rightarrow \quad x \leq 3 \\ x \neq 1 \end{cases}$$

$$\mathbb{D} = [0, 1) \cup (1, 3]$$

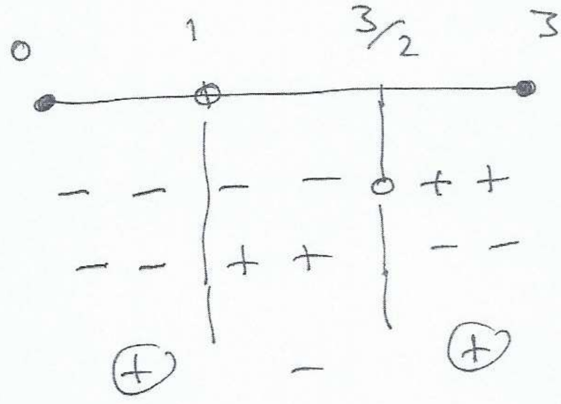
$$\frac{\sqrt{x} - \sqrt{3-x}}{x-1} > 0$$

$$N(x) = \sqrt{x} - \sqrt{3-x} > 0 \Rightarrow \quad \sqrt{x} > \sqrt{3-x}$$

$$x > 3 - x$$

$$2x > 3 \Rightarrow x > \frac{3}{2}$$

$$D(x) = x - 1 > 0 \Rightarrow x > 1$$



$$f(x) > 0 \qquad \forall x \in [0, 1) \cup (\tfrac{3}{2}, 3]$$

$$f(x) = 0 \quad \text{per} \quad x = \frac{3}{2}$$

$$f(x) < 0 \quad \text{per} \quad x \in (1, \tfrac{3}{2})$$